St John Baptist De La Salle Catholic School, Addis Ababa

Homework 1 Solutions

3rd Quarter

Aaron GK

February 27, 2023

Notes, and use of other aids is allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work. **Cheating or indications of cheating and similar answers will be punished accordingly**.

Information

- The homework is due on **Wednesday**, **February 22**.
- You should Work on it **in groups** and consult me if you have any questions. Cheating within groups is unacceptable.
- For purposes of neatness and simplicity of grading, you should do the homework on an **A-4 paper**.

Questions

1. For vectors $\vec{A}$=-$\hat{i}$-4$\hat{j}$+6$\hat{k}$ and $\vec{B}$=3$\hat{i}$-7$\hat{j}$-3$\hat{k}$, calculate:

   (A) $\vec{A}+\vec{B}$

   Answer

   To add the vectors, we add the similar components as we would do with polynomial expressions:

   $$\vec{A}+\vec{B}=(-\hat{i}-4\hat{j}+6\hat{k})+(3\hat{i}-7\hat{j}-3\hat{k})$$
   $$\vec{A}+\vec{B}=-2\hat{i}-11\hat{j}+3\hat{k}$$

   (B) $2\vec{A}-\vec{B}$

   Answer

   We compute the following just as we did in part (A), but we have to multiply each component of $\vec{A}$ by 2.

   $$2\vec{A}-\vec{B}=2(-\hat{i}-4\hat{j}+6\hat{k})-(3\hat{i}-7\hat{j}-3\hat{k})=-2\hat{i}-8\hat{j}+12\hat{k}-(3\hat{i}-7\hat{j}-3\hat{k})$$
   $$2\vec{A}-\vec{B}=-5\hat{i}-\hat{j}+15\hat{k}$$

   (C) Find the magnitudes of $\vec{A}+\vec{B}$ and $2\vec{A}-\vec{B}$ and the unit vectors in their directions.

   Answer

   For $\vec{A}+\vec{B}$,

   The magnitude is given by:

   $$|\vec{A}+\vec{B}|=\sqrt{(-2)^2+(-11)^2+3^2}=\sqrt{134}$$

   The unit vector in the direction of $\vec{A}+\vec{B}$ is given by the following:

   $$\frac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|}=\frac{-2\hat{i}-11\hat{j}+3\hat{k}}{\sqrt{134}}$$

   You can do the same for $2\vec{A}-\vec{B}$.

-

2. Find the unit vector of direction for the following vector quantities:

   (I) $\vec{F}=2\hat{i}-3\hat{j}+10\hat{k}$

   Answer

   We first need to find the magnitude of $\vec{F}$

   $$|\vec{F}|=\sqrt{(2)^2+(-3)^2+(10)^2}$$
   $$|\vec{F}|=\sqrt{(2)^2+(-3)^2+(10)^2}=\sqrt{113}$$

   To find the unit vector $\hat{F}$;

   $$\hat{F}=\frac{\vec{F}}{|\vec{F}|}=\frac{2\hat{i}-3\hat{j}+10\hat{k}}{\sqrt{113}}$$

(II) $\vec{B} = 5\hat{i} - 7\hat{j} + 15\hat{k}$

(III) $\vec{V} = 10\hat{i} - 7\hat{k}$
Compute II & III similarly.

3. A two dimensional force vector has a magnitude of 30N and is acting at an elevation angle of 37 degrees with respect to the origin. Write the force vector in its component form.
Answer
We first need to resolve the vector into its components. We have vertical(along y-axis) and horizontal(along x-axis) components:

$$\vec{A}_x = |\vec{A}| \cos\theta \hat{i}$$
$$\vec{A}_x = 30N \times \cos 37^0 \hat{i} = 24N\hat{i}$$
$$\vec{A}_y = |\vec{A}| \sin\theta \hat{j}$$
$$\vec{A}_y = 30N \times \sin 37^0 \hat{j} = 18N\hat{j}$$
$$\vec{A} = \vec{A}_x + \vec{A}_y$$
$$\vec{A} = (24\hat{i} + 18\hat{j})N$$

4. For the two points in the Cartesian plane A(2, 8) and B(-3, 5), find their respective position vectors and calculate the magnitude of those vectors.
Answer

$$\vec{A} = 2\hat{i} + 8\hat{j}$$
$$\vec{B} = -3\hat{i} + 5\hat{j}$$

5. Show that the vectors $\vec{A} = \frac{10}{3}\hat{i} - 6\hat{j}$ and $\vec{B} = \frac{6}{5}\hat{i} + \frac{2}{3}\hat{j} + 10\hat{k}$ are perpendicular.
Answer
To show that the two vectors are perpendicular, we can just show that their dot product is 0.

$$\vec{A} \cdot \vec{B} = (\frac{10}{3}\hat{i} - 6\hat{j}) \cdot (\frac{6}{5}\hat{i} + \frac{2}{3}\hat{j} + 10\hat{k})$$
$$\vec{A} \cdot \vec{B} = \frac{10}{3}(\frac{6}{5}) - 6(\frac{2}{3}) = 0$$

6. Find the angle between the vectors **A** and **B** in question 1.

7. For any vector $\vec{A}$, what is $\vec{A} \cdot \vec{A}$?
Answer

$$\vec{A} \cdot \vec{A} = |\vec{A}||\vec{A}| \cos 0^0 \text{ since the angle between a vector and itself is 0 degrees}$$
$$\vec{A} \cdot \vec{A} = |\vec{A}|^2$$

Advanced Problems

8. What is the projection of the force vector G =($\hat{i}$-5$\hat{j}$+3$\hat{k}$)N along the force vector H =(-3$\hat{i}$+$\hat{j}$)N?
Answer
The projection of a vector $\vec{G}$ along $\vec{H}$ is the component of $\vec{G}$ that is in the same direction as $\vec{H}$. Thus:

$$\text{proj}_{\mathbf{H}}^{\mathbf{G}} = |\vec{G}| \cos\theta \hat{H}$$
$$\text{proj}_{\mathbf{H}}^{\mathbf{G}} = \frac{\vec{G} \cdot \vec{H}}{|\vec{H}|} \frac{\vec{H}}{|\vec{H}|}$$
$$\text{proj}_{\mathbf{H}}^{\mathbf{G}} = \frac{\vec{G} \cdot \vec{H}}{|\vec{H}|^2} \vec{H}$$

First, we find the dot product between the two vectors:

$$\vec{G} \cdot \vec{H} = -3 - 5 = -8$$
$$|\vec{H}| = \sqrt{(-3)^2 + (1)^2} = \sqrt{10}$$
$$\text{proj}_{\mathbf{H}}^{\mathbf{G}} = \frac{-8}{\sqrt{10}}(-3\hat{i} + \hat{j})N$$