St John Baptist De La Salle Catholic School, Addis Ababa

Homework 2 Solution

3rd Quarter

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Notes, and use of other aids is allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work. **Cheating or indications of cheating and similar answers will be punished accordingly**.

Information

- The homework is due on **Friday**, **March 3**.
- You should Work on it **in groups** and consult me if you have any questions. Cheating within groups is unacceptable.
- For purposes of neatness and simplicity of grading, you should do the homework on an **A-4 paper**.

Questions

1. When is the cross product used? How is it different from the dot product? What are the applications of the cross product?
   Answer:

   Cross product is used in situation in which the product of the vectors results in a vector. It is mainly different from the dot product in that the result in a dot product is always a scalar, whereas a vector product results in a vector. We use the cross product in many situations involving product of vectors giving vectors; it is applicable while finding the directions of many essential vectors such as the force & torque.

2. Let $\vec{A} = -\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ and $\vec{D} = 9\hat{\imath} - H\hat{\jmath} + 5\hat{k}$. For what value(s) of H are the vectors A and D perpendicular?
   Answer:
   We know that the dot product of two vectors is 0 when they are perpendicular. If the vectors $\vec{A}$ and $\vec{D}$ are perpendicular, $\vec{A} \cdot \vec{D} = 0$

$$\vec{A} \cdot \vec{D} = 0$$
$$(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) \cdot (9\hat{\imath} - H\hat{\jmath} + 5\hat{k}) = 0$$
$$-9 - 2H + 25 = 0 \implies -2H = -16$$
$$H = 8$$

3. After you find the value of H, find $\vec{A} \times \vec{D}$, $|\vec{A} \times \vec{D}|$, and a unit vector perpendicular to both $\vec{A}$ and $\vec{D}$.
   Answer:

$$\vec{A} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) \times (9\hat{\imath} - 8\hat{\jmath} + 5\hat{k})$$

   To calculate the cross product, we simply can use the matrix method

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 2 & 5 \\ 9 & -8 & 5 \end{vmatrix}$$

$$\vec{A} \times \vec{D} = \hat{\imath}(10 + 40) - \hat{\jmath}(-5 - 45) + \hat{k}(8 - 18)$$
$$\vec{A} \times \vec{D} = 50\hat{\imath} + 50\hat{\jmath} - 10\hat{k}$$
$$|\vec{A} \times \vec{D}| = \sqrt{50^2 + 50^2 + (-10)^2} = \sqrt{5100} = 10\sqrt{51}$$

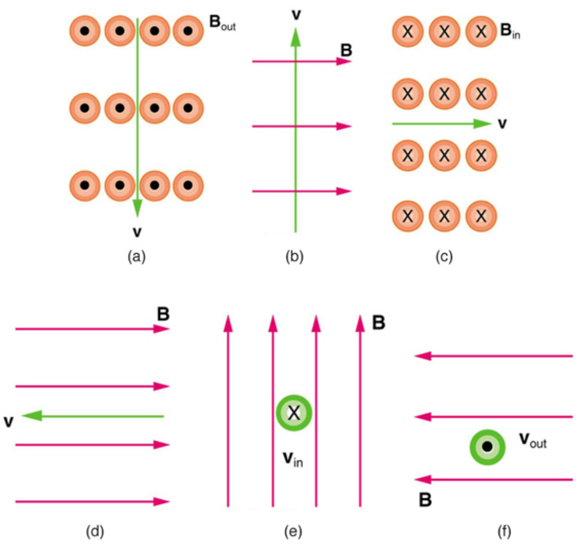
   To find a unit vector perpendicular to both $\vec{A}$ and $\vec{D}$, we can just find the unit vector in the direction of $\vec{A} \times \vec{D}$. Let us, for example, name that vector $\hat{n}$;

$$\hat{n} = \frac{\vec{A} \times \vec{D}}{|\vec{A} \times \vec{D}|} = \frac{50\hat{\imath} + 50\hat{\jmath} - 10\hat{k}}{10\sqrt{51}} = \frac{5\hat{\imath} + 5\hat{\jmath} - \hat{k}}{\sqrt{51}}$$

4. For two vectors $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$, show that $(\vec{A}\times\vec{B})\cdot\vec{B} = 0$
Answer:

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k}$$
$$(\vec{A}\times\vec{B})\cdot\vec{B}$$
$$((A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{\jmath})$$
$$(A_yB_z - A_zB_y)B_x + (A_zB_x - A_xB_z)B_y + (A_xB_y - A_yB_x)B_z$$
$$(A_yB_zB_x - A_zB_yB_x) + (A_zB_xB_y - A_xB_zB_y) + (A_xB_yB_z - A_yB_xB_z)$$

We can see that the above terms cancel out to give 0.



5. For the figure above, find the direction towards which a positive charge would be moving in the various magnetic fields.
Answer:

(a) $q\vec{v}\times\vec{B} \implies -\hat{\jmath}\times\hat{k} = -\hat{\imath}$: the charge will move to the left.
(b) $q\vec{v}\times\vec{B} \implies \hat{\jmath}\times\hat{\imath} = -\hat{k}$: the charge will move out of the page.
(c) $q\vec{v}\times\vec{B} \implies \hat{\imath}\times -\hat{k} = \hat{\jmath}$: the charge will move upwards.
(d) $q\vec{v}\times\vec{B} \implies -\hat{\imath}\times\hat{\imath} = 0$: the charge **will keep moving** to the left.
(e) $q\vec{v}\times\vec{B} \implies -\hat{k}\times\hat{\jmath} = \hat{\imath}$: the charge will move to the right.
(f) $q\vec{v}\times\vec{B} \implies \hat{k}\times -\hat{\imath} = -\hat{\jmath}$: the charge will move downwards.

6. A charged particle of mass twice that of an electron and a charge of -3.2×10^{-19}C is moving about a circular trajectory of radius 20cm in a uniform 3.5×10^{3}G magnetic field that is perpendicular to the velocity of the charge. What is the velocity of the charge?
Answer:

$$qvB\sin\theta = \frac{mv^2}{r}$$
$$v = \frac{qBr\sin\theta}{m}$$
$$v = \frac{3.2\times10^{-19}C\times3.5\times10^{-1}T\times0.2m}{2\times9.11\times10^{-31}kg}$$

7. An electron moving at a speed of 0.7c through a magnetic field of 2.0T experiences a magnetic force of 2.2×10^{-14}N. What is the angle between the electron's velocity and the magnetic field?
Answer:

$$F = qvB\sin\theta$$
$$\sin\theta = \frac{F}{qvB}$$
$$\sin\theta = \frac{2.2\times10^{-14}N}{1.6\times10^{-19}C\times0.7\times3\times10^{8}m/s\times2.0T} = \frac{2.2\times10^{-14}N}{3.2\times2.1\times10^{-11}N} = \frac{2.2}{3.2\times2.1}\times10^{-3}$$
$$\theta = \sin^{-1}\left(\frac{2.2}{3.2\times2.1}\times10^{-3}\right)$$

Advanced Problems

8. A uniform magnetic field of magnitude 1.2 T is directed along the negative y - axis. An electron moving at a speed of $0.2c$ makes an angle of 60^0 with the y - axis. Answer the following questions.

   (I) What is the expected trajectory of the electron?
   Answer:
   The expected trajectory is a helix. That is because the perpendicular component of the velocity will make the electron travel about a circle while the parallel component keeps the electron in its original path. The superposition of the two motions results in a helical trajectory.

   (II) Calculate the radius & pitch of the trajectory.
   Answer:

$$r = \frac{mv}{qB\sin\theta}$$

$$r = \frac{9.11 \times 10^{-31}kg \times 0.2 \times 3 \times 10^8 m/s}{1.6 \times 10^{-19}C \times 1.2T \times \sin 60^0}$$

   To calculate the pitch, first let's look at what it is. The pitch is the distance traveled parallel to the magnetic field B in one revolution. Thus,

$$p = v_{\parallel}T \text{ where T is the time needed to complete a revolution}$$

   To calculate the pitch, we first need to calculate the time it takes to complete a revolution.

$$v_{\perp} = \frac{2\pi R}{T}$$

$$T = \frac{2\pi R}{v_{\perp}}$$

   Then, we can calculate the pitch as follows:

$$p = v_{\parallel}(\frac{2\pi R}{v_{\perp}})$$

$$p = v\cos\theta(\frac{2\pi R}{v\sin\theta}) = \cos\theta(\frac{2\pi R}{\sin\theta})$$

$$p = (\frac{2\pi R}{\tan\theta})$$

$$p = \frac{2\pi \times \dfrac{9.11 \times 10^{-31}kg \times 0.2 \times 3 \times 10^8 m/s}{1.6 \times 10^{-19}C \times 1.2T \times \sin 60^0}}{\tan 60^0}$$