Further Notes on Capacitance & Resistance

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Capacitance & Factors affecting it

$C = \kappa\epsilon_0 \frac{A}{d}$

Solution
We can express the electric field strength in in terms of charge and area as follows:

$$E = \frac{Q}{A\varepsilon_0}$$

We have seen that we can express the potential difference between charged plates as a product of the field and the distance between them:

$$V = Ed$$

$$V = \frac{Q}{A\varepsilon_0}d = \frac{Qd}{A\varepsilon_0}$$

However, capacitance is defined as follows:

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\frac{Qd}{A\varepsilon_0}}$$

$$C = \frac{\varepsilon_0 A}{d}$$

If we add a dielectric, the field decreases by a factor of κ

$$E_f = \frac{E}{\kappa}$$

$$V_f = \frac{Vd}{\kappa}$$

$$V_f = \frac{\frac{Qd}{A\varepsilon_0}}{\kappa} = \frac{Qd}{A\varepsilon_0\kappa}$$

Thus, the final capacitance after a dielectric has been added is given as follows:

$$C = \frac{Q}{V_f}$$

$$C = \frac{Q}{\frac{Qd}{A\varepsilon_0\kappa}}$$

$$C_f = \frac{\kappa\varepsilon_0 A}{d}$$

Resistance & Factors affecting it

Ohm's Law at the microscopic level can be described in terms of the resistivity and electric field as follows:

$$J = \frac{E}{\rho}$$

Where **J** is the current density vector and **E** is the electric field vector. The current density vector is the time rate of flow of charges through a given cross-sectional area. Thus, we define current density as follows:

$$J = \frac{I}{A}$$

We know that the microscopic definition of Ohm's Law for Ohmic substances is that the current through is directly proportional to the potential difference across it - ($I \propto V$) We thus define resistance R as follows:

$$I \propto V$$

$$I = \frac{1}{R} \times V$$

Here, the constant R is called the resistance of the conductor. We can express R in terms of V and I as follows:

$$R = \frac{V}{I}$$

To see the factors affecting resistance, let's try to express the current in terms of voltage.

$$J = \frac{I}{A}, \text{ but we also know that } J = \frac{E}{\rho}, \text{ thus:}$$

$$\frac{I}{A} = \frac{E}{\rho}$$

We can express the electric field inside a conductor in terms of the potential difference across it and its length:

$$V = El \implies E = \frac{V}{l}, \text{ replacing this expression E of in the equation above, we get}$$

$$\frac{I}{A} = \frac{\frac{V}{l}}{\rho} \implies \frac{I}{A} = \frac{V}{l\rho}$$

We know that we can express the voltage as a function of the resistance and current: $V = IR$. Plugging in this value of V into the equation above, we get:

$$\frac{I}{A} = \frac{IR}{l\rho}$$

The current cancels out and we get:

$$\frac{1}{A} = \frac{R}{l\rho}, \text{ thus we can express the resistance in terms of the other quantities}$$

$$R = \frac{\rho l}{A}$$

Thus, this tells us that the resistance of an Ohmic material has a direct relationship with ρ and l while it has an inverse relationship with the area.